Has Deep Karst a Fractal Behaviour?

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Abstract

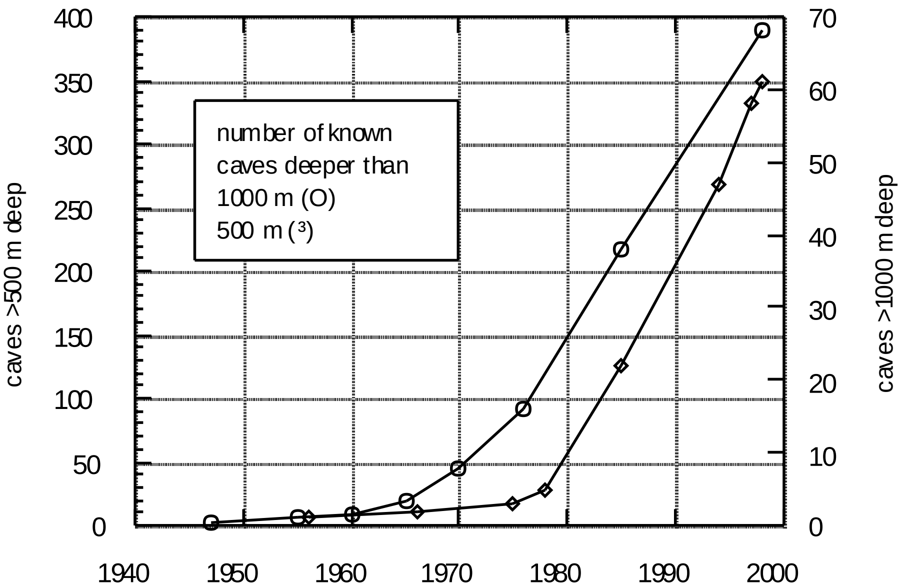
In the last 50 years the cavers explorations have increased exponentially the number of known caves enabling to built a consistent database. We have used the SIS-INRIA catalogue to analyse the distributions of length and depth of world caves. The cave length appears to be scale invariant that indicates fractal behaviour of genetic processes. The cave depth has a scale length of roughly 270 meters, probably connected with mountain scale dimensions.

Speleology is quite old but the modern approach was born in the last half of XX century. The length of all the conduits explored in Italy now amounts to 2300 km, and we may guess a world figure around 15000 km, but the explorative work has just begun. The table below shows the density of conduits in some large and well-explored caves.

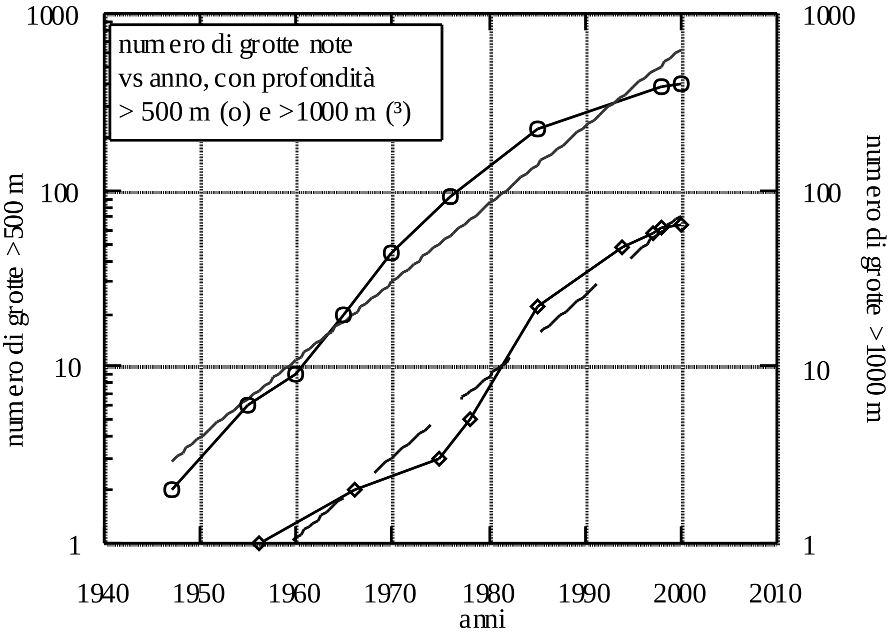
Cave system	Surface[km^2]	Lenght [km]	Vol. density[km\km^3]	Surf. density [km/km^2]
Jewel	6	120	120	20
Mammoth	80	550	34	7
Pierre st.m	20	50	1.7	2.5
Hyournedo	10	90	9	9
Piaggia bella	4	40	10	10
Col delle erbe	1.8	14	8	8
Corchia	3	50	14	16

In the table is possible to see that the surface density of caves is quite regularly, pointing to a ratio around 10 km of conduits per km^2 of field. We may suppose that the total length of caves is of the order of many millions of kilometres, hundredths times more than the explored up to now.

We may have a confirmation of this looking the distribution of number of known deep caves vs. time:



The graph shows a steep rise, but it does not show a well defined behaviour or a clear law. If we redraw it on semi-log scale we obtain:



It is possible to see that the increase is almost linear on time versus the logarithm of the number of caves, meaning that the number of explored caves increases geometrically, but it is possible to see a slowing rate in the lasts 15 years.

The next table shows the time required to double the number of known deep caves:

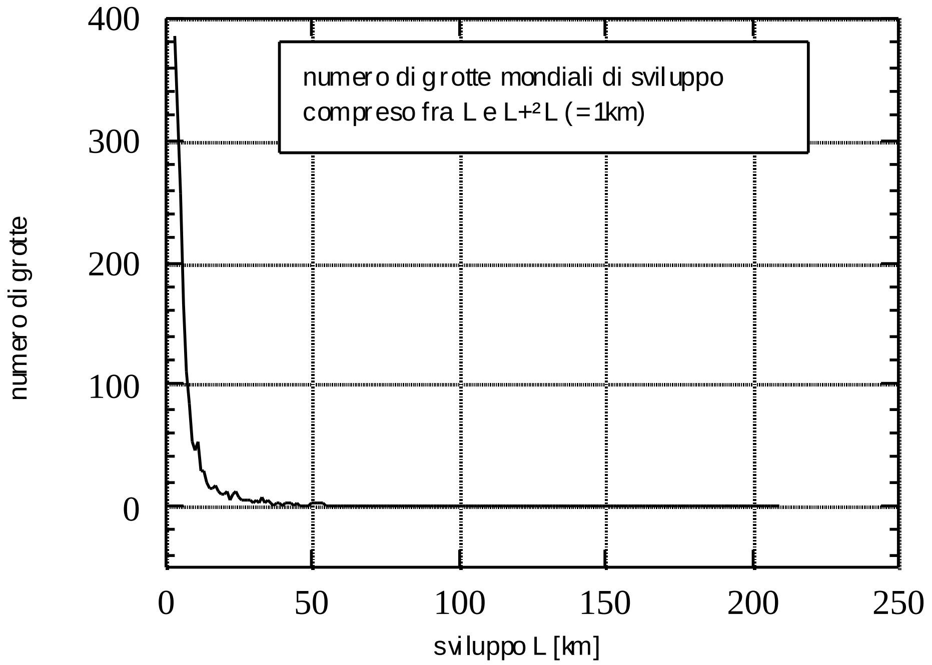
Time to double	1960-1985	1985-2000
>500 metres	5.5 y	17 y
>1000 meters	5.5 y	9 y

Exponential increases like these shows that we are very far from a complete knowledge of the deep karstic world.

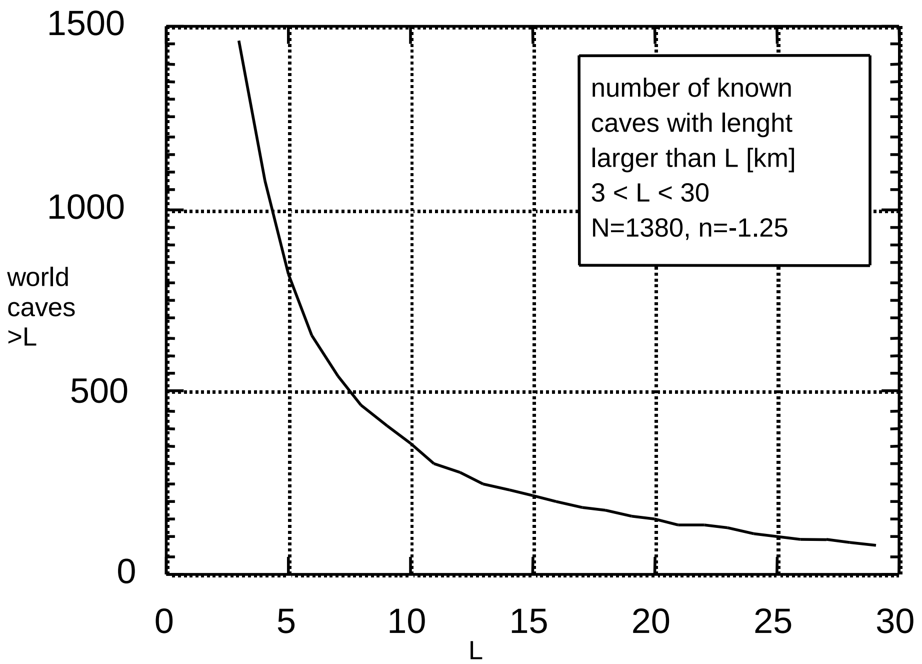
Let us discuss some features of this world as a whole.

In recent years large cave archives have been produced. We have considered here the deepest and longer caves list given by Eric Madeleine (SIS-INRIA, Sophia Antipolis, France) that consider a sample of 1400 long and 1000 deep caves, to extract some details on their distribution.

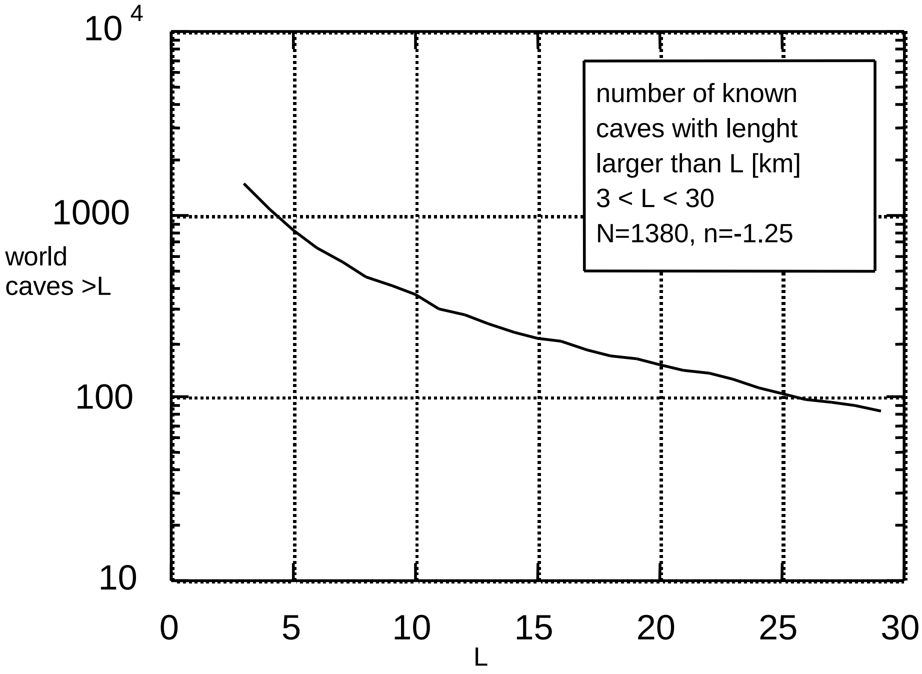
At first we consider the number of known caves longer than a fixed length L, i.e. the cumulative distribution of the caves length.



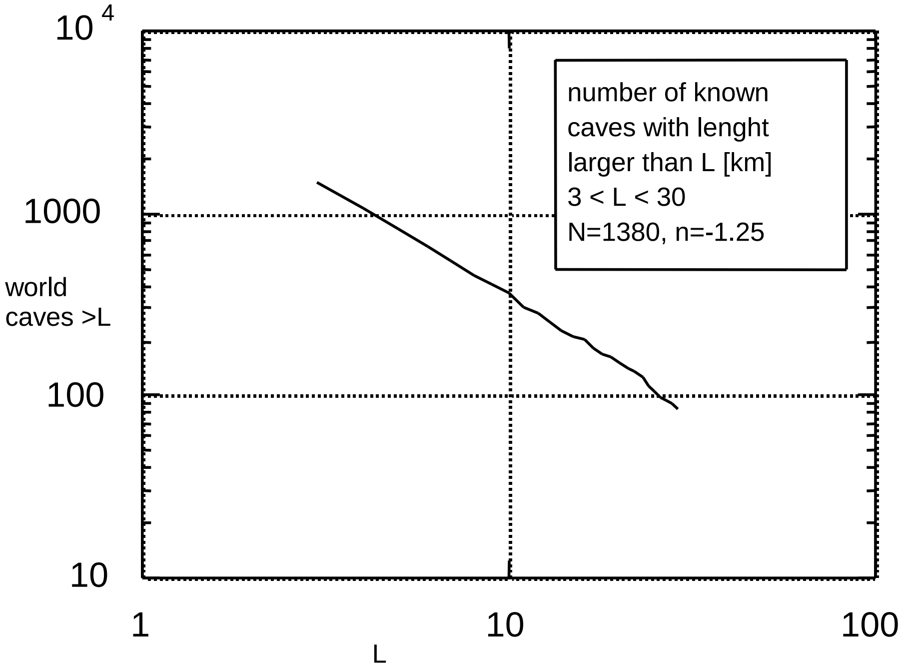
Let us focus our analysis in the length range up to 30 km:



Again, it seems a good idea to analyse it with semi-log scale.



The graph above shows us that the cumulative distribution is not exponential. If we use now the log-log scale we discover something:



As the cumulative distribution is a straight line, we conclude that there is a linear law connecting log(L) and log(N):

$$\log(N) = k\log(L) = \log(L^k)$$

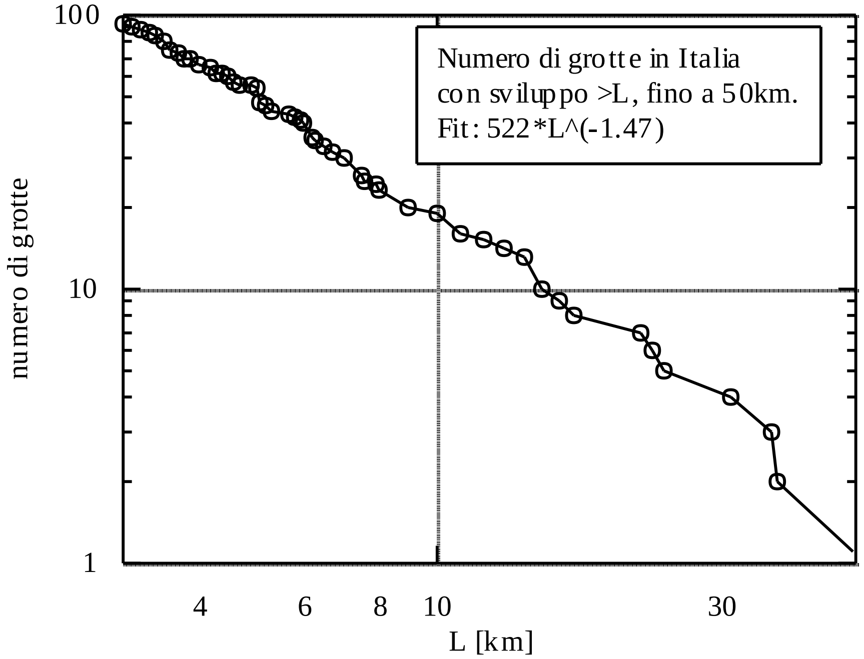
$$N = AL^n$$

Than there is, a power law connecting the lengths of the caves.

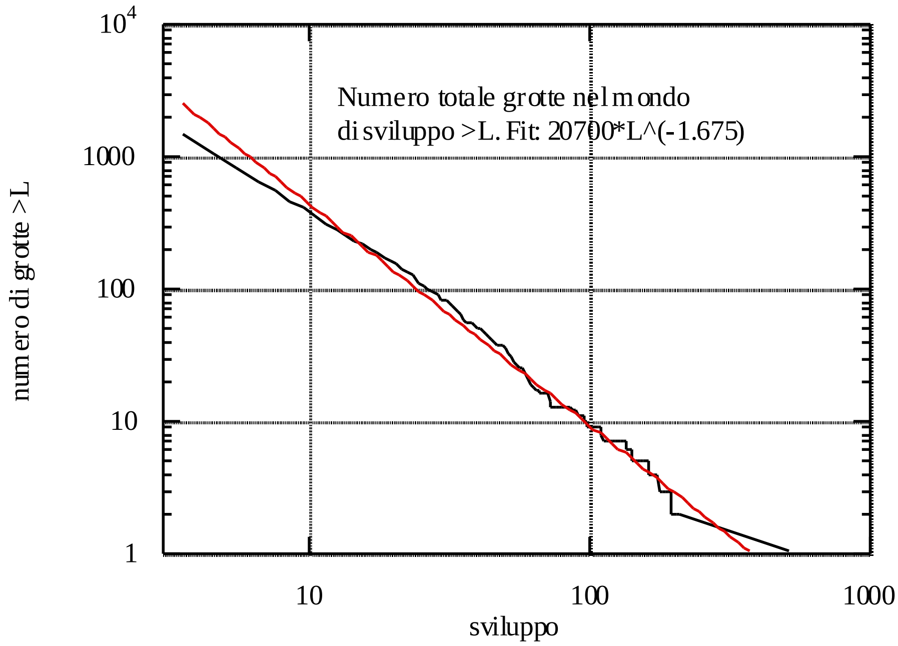
A similar law in physical systems describes processes scale-invariant, and is generally given by hidden fractal behaviour of generating processes (TURCOTTE, 1992).

The spectral index n is –1.25 for cave lengths between 3 and 30 km.

Let us check if the spectral index is related to some specific type of karst, analysing the data of a single country: we have used the Italy a sample.



The power law behaviour is unchanged, only the spectral index shows a small change. The global distribution is:

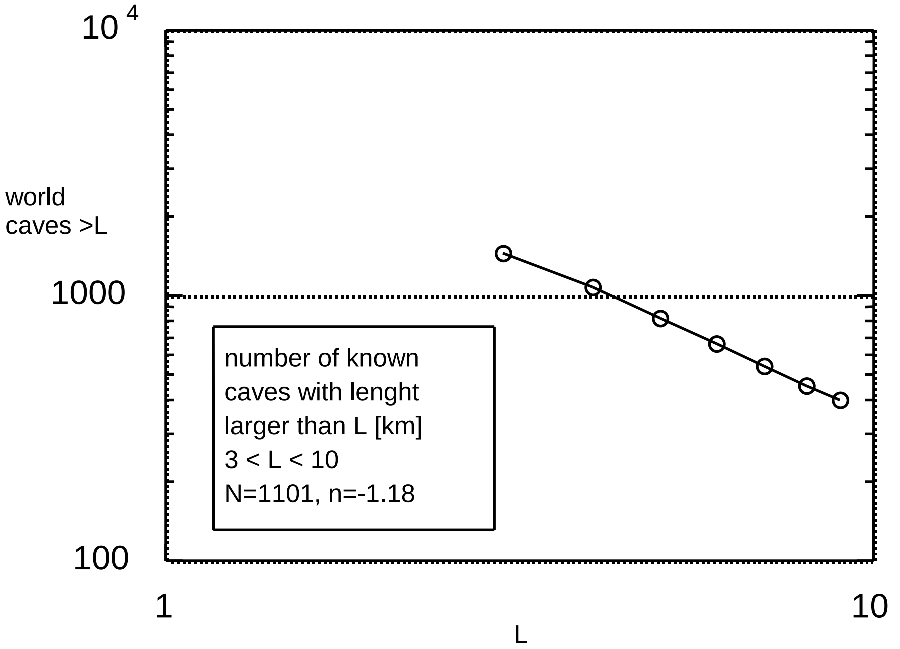


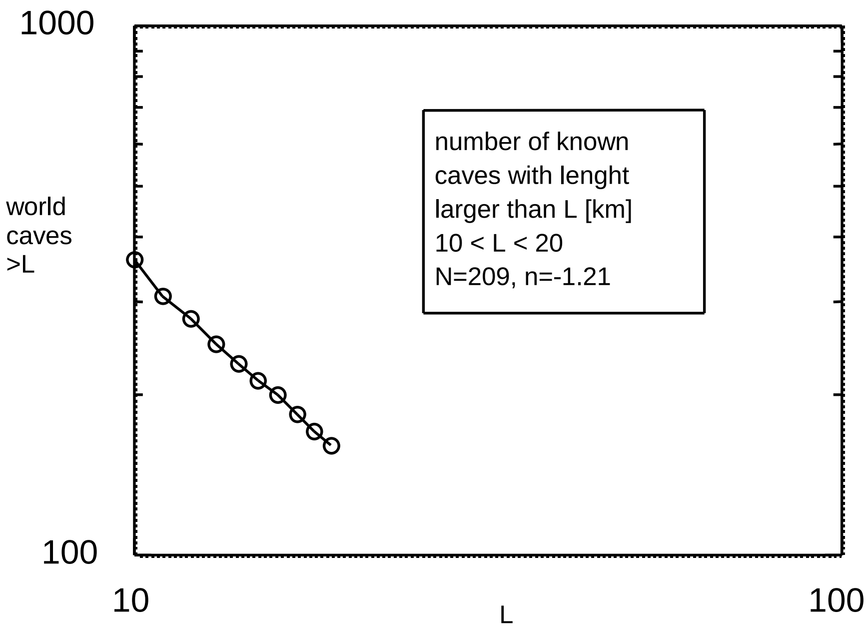
The global course is linear, nevertheless showing important fluctuations, mainly in the higher part, where the statistic is very poor.

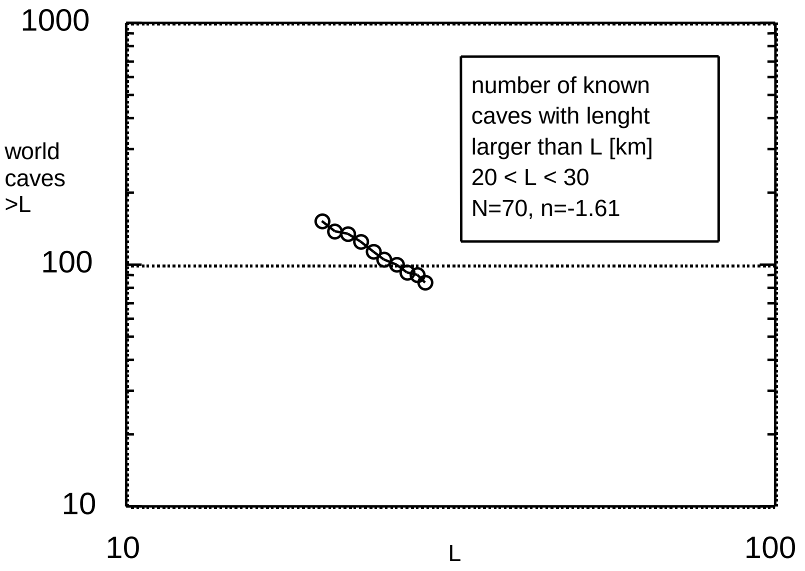
Obviously, this distribution is given by a large amount of different contributing processes. Some of them are physical, and typically resulting in a fractal behaviour: fragmentation of the sedimentary fields, fragmentation in different hydrological systems, fragmentation of rock and so on. Other are not connected with the geological system but with factors related to their knowledge: number of entrances, practical difficulties, local history of researches and so on.

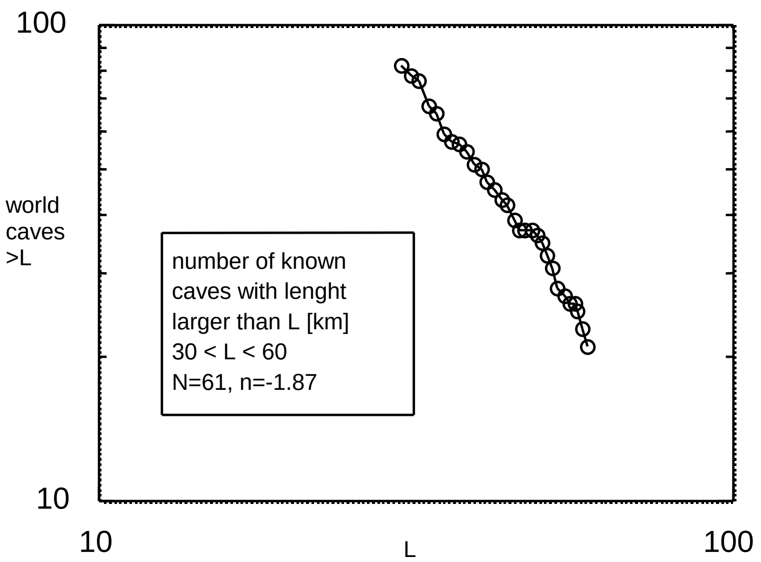
It is reasonable to look for larger details, considering different intervals of the cumulative distribution. We have divided the 3-560 km in different ranges.

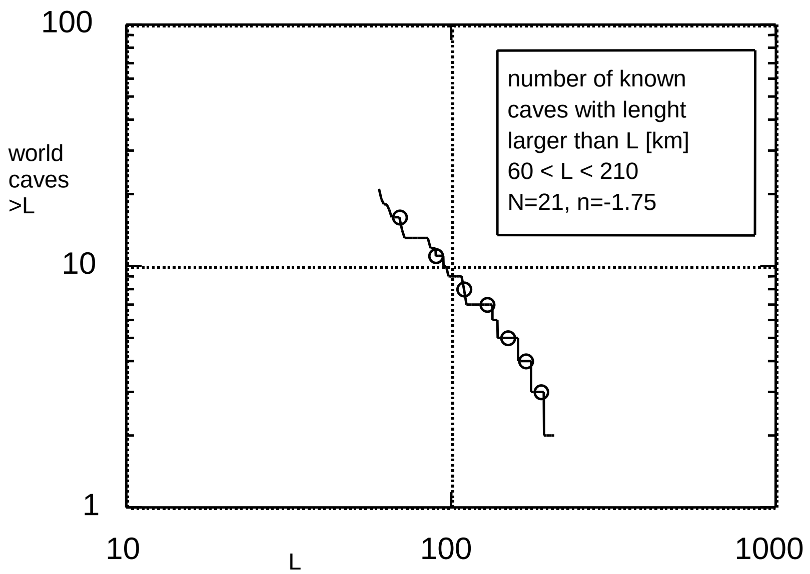
The first 3≤L<10 km, is probably the most representative due to its large statistic (1100 elements) and the world diffusion of involved caves. The spectral index is –1.18.






1000
world
caves
>L
100
10
10
L
100
number of known
caves with lenght
larger than L [km]
20 < L < 30
N=70, n=-1.61


100
world
caves
>L
10
10
L
100
number of known
caves with lenght
larger than L [km]
30 < L < 60
N=61, n=-1.87


100
world
caves
>L
10
1
10
L
100
1000
number of known
caves with lenght
larger than L [km]
60 < L < 210
N=21, n=-1.75

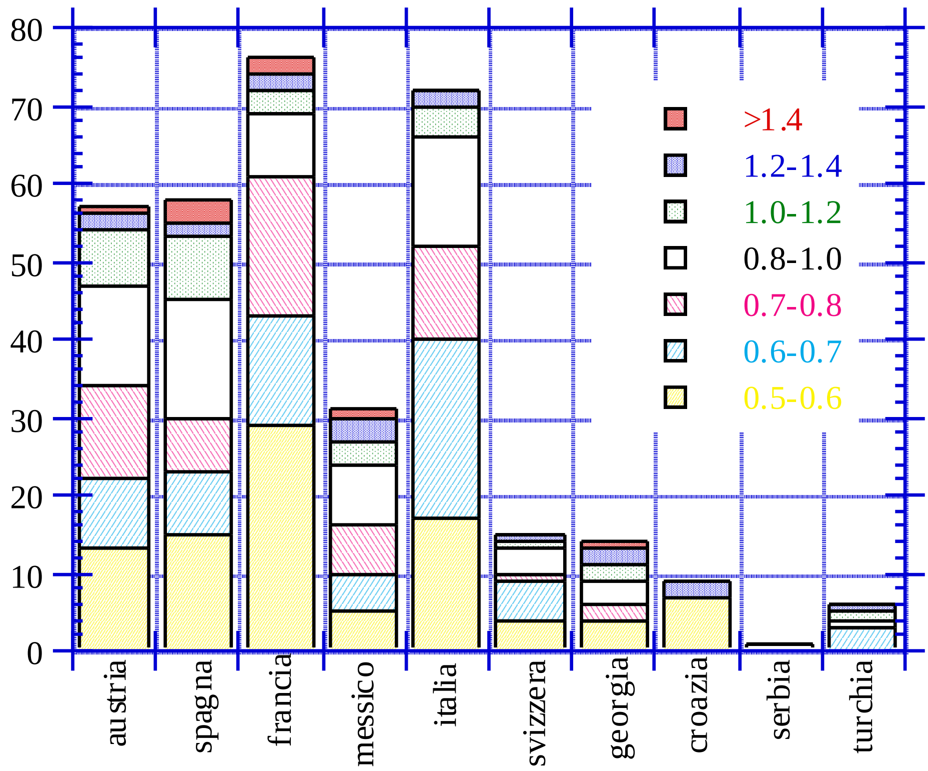
The other ranger shows a strong increase of spectral index, despite their quickly reducing statistics. The last decrease does not appear to be statistically significant, and would be stronger if Mammoth-Flint Ridge caves were included.

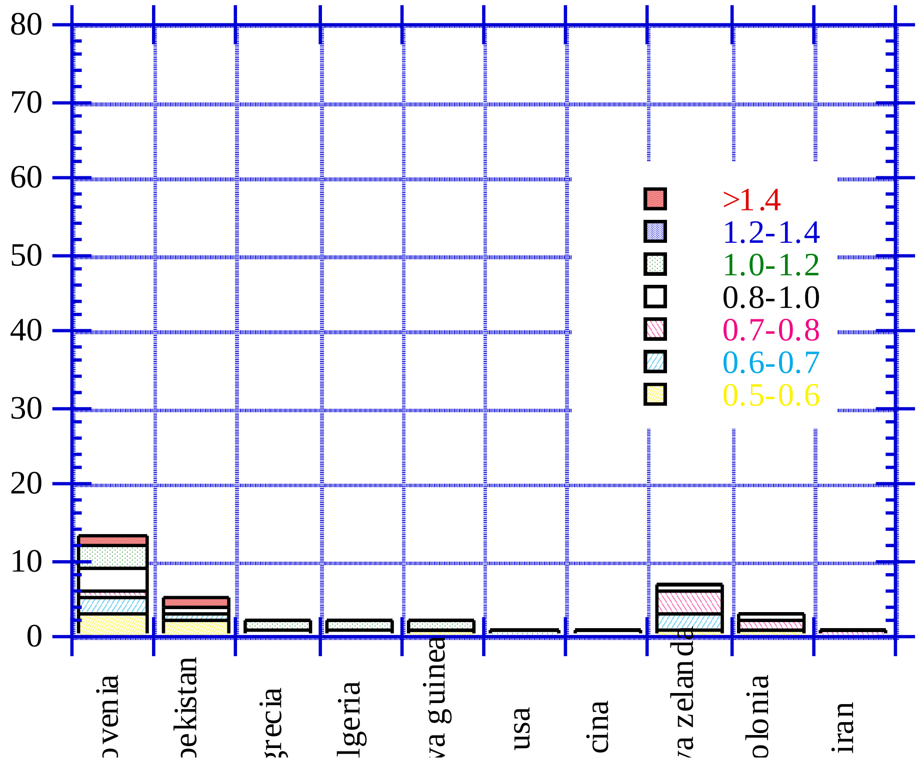
Range	Elements	Spectral index n
3≤L<10	1101	-1.19
10≤L<20	209	-1.21
20≤L<30	70	-1.71
30≤L<60	61	-1.87
60≤L<210	21	-1.75

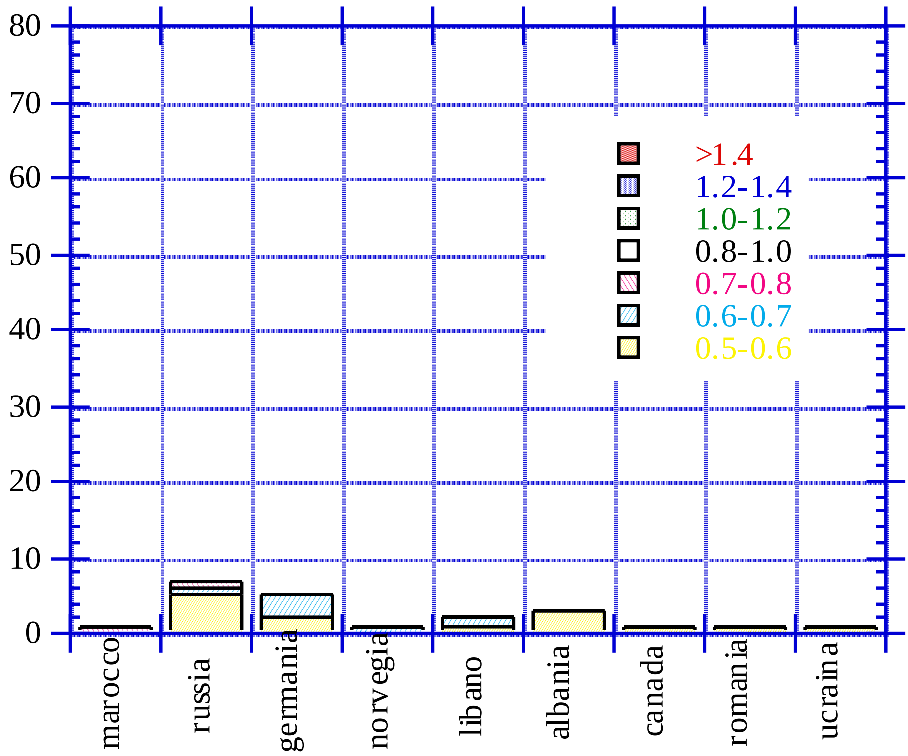
Let us consider now the deep caves. Physically the meaning of the cave depth (altitude difference between the highest and lowest points of caves) does not appear to be very pregnant because the loss of information is too strong.

If we consider the world distribution of deep caves it is also evident that the "deep caves" information come from researches that are not in worldwide scale.

On the almost 350 caves deeper than 500 meters, 22% are in France, 21% in Italy, 17% in Spain and Austria, that is more than ¾ of deep caves are in four countries that represents less than 0.5% of lands.

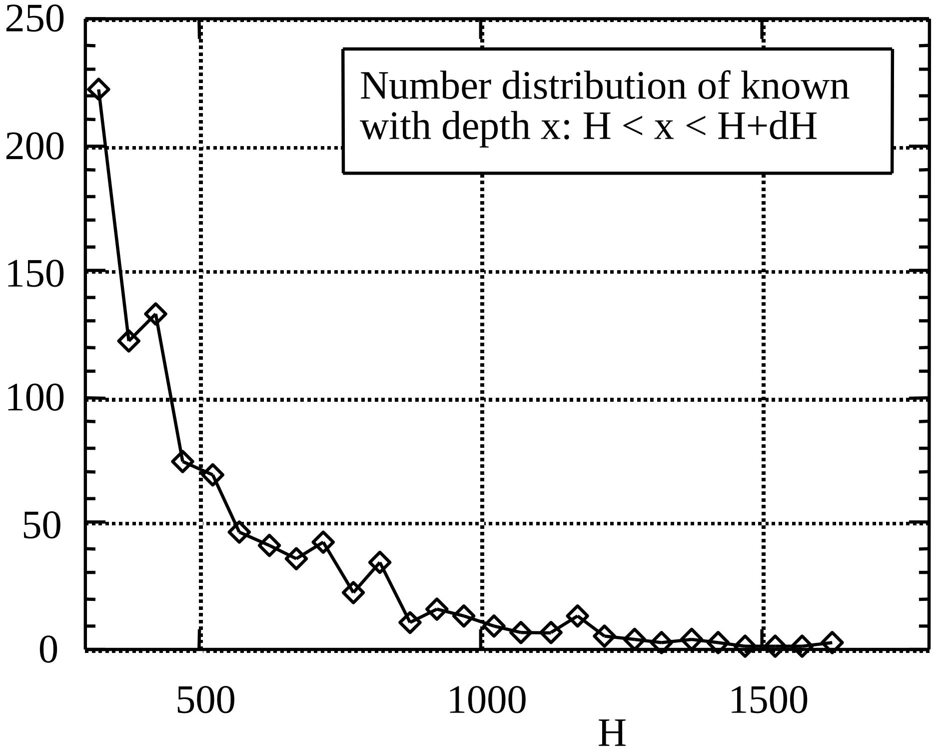


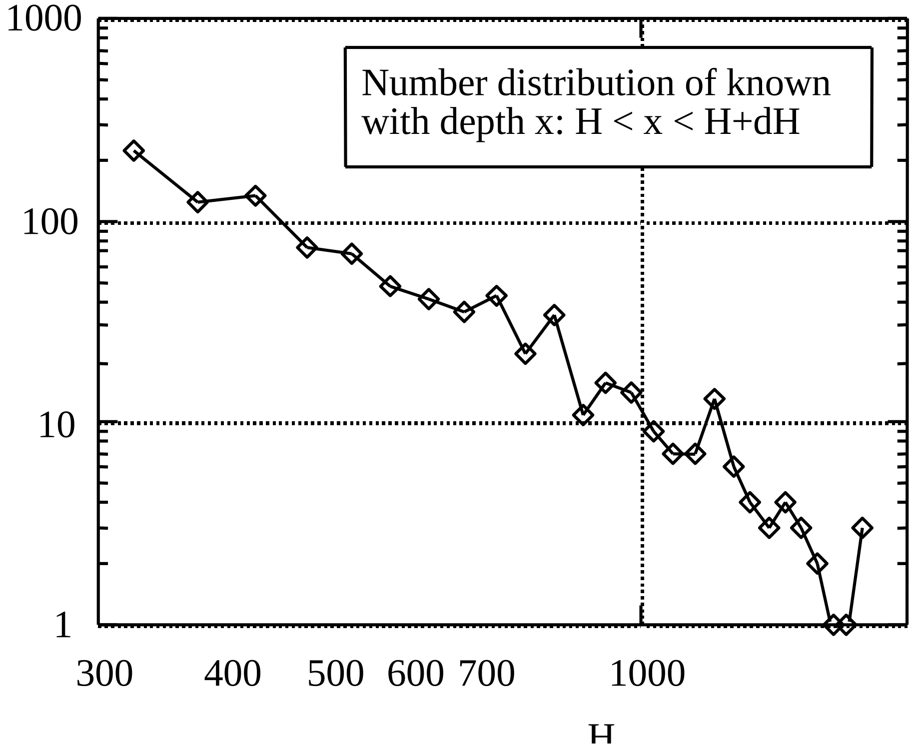


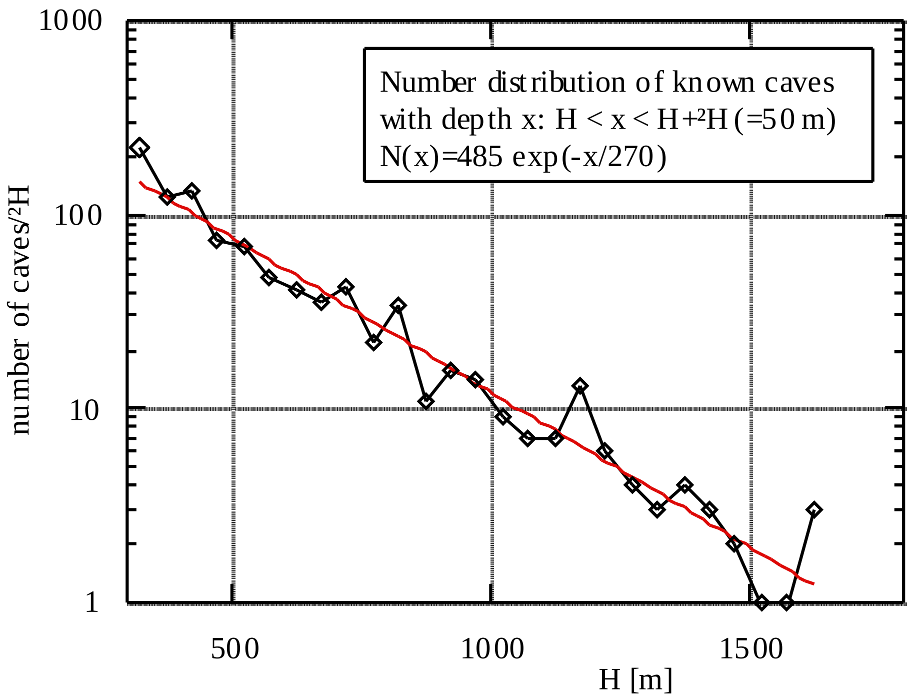


The analysis of the distribution is very interesting. We are going to do it with the differential spectra, that is the number of caves of depth x laying in the interval H≤x<ΔH. We have chosen a ΔH step of 50 meters.

The distribution on linear scale does not give interesting results. Also the redrawing on a log-log scale does not show a hidden law.


250
200
150
100
50
0
500
1000
1500
H
Number distribution of known
with depth x: H < x < H+dH


1000
100
10
1
300
400
500
600
700
1000
H
Number distribution of known
with depth x: H < x < H+dH



Analysing it on a semi-log scale we obtain an impressive result.

The straight line shows an exponential behaviour with a typical length, showing that the cave depths are not scale invariant. So, it really exists a scale law for the cave depths, and in fact one can understand the typical length involved in a cave, just looking at a sketch of its vertical section. The fit of the data is:

$$N = A\exp\left(-\frac{x}{p_g}\right)$$

$$A = 4080$$

$$p_g = 270\text{m}$$

This distribution is Poissonian-like and may be interpreted as follows: exploring a cave, in any mountain, it is possible to go down one meter more with a probability equal to 1/240…

The real meaning is probably that it corresponds to the average karstic limestone thickness, that is, the inner dimension scale of karstic phenomena could be directly connected with the external scale of the mountain dimensions.

Conclusions

The cave statistic becomes to be sufficient to perform global analysis. The cave length appear to be scale invariant appointing to a fractal behaviour of genetic processes, but the statistics have to be extended to small caves.

The cave depth has a length scale of roughly 270 meters and is probably connected with mountain scale dimensions.

Bibliography

Turcotte, Fractals and chaos in geology and geophysics, Cambridge Un. Press, 1992